Discovering Math: Problem Solving
Teacher’s Guide

Grade Level: 6–8  |  Curriculum Focus: Mathematics  |  Lesson Duration: Three class periods

Program Description
Discovering Math: Problem Solving — From determining pertinent information to trying and comparing multiple strategies, this video uses practical examples of logic and reasoning to illustrate how mathematics is used to solve real-world problems.

Lesson Plan
Student Objectives
- Use mathematical operations, problem-solving strategies, and critical thinking to solve a complex problem.
- Research a chosen weather phenomenon and a problem related to that weather phenomenon.
- Make a generalization and conjectures about the chosen weather phenomenon based on research.
- Use inductive and deductive reasoning to formulate a problem and create a solution.
- Use various approaches (make a table, create a model, or draw a diagram) to solve a problem.
- Identify limitations of a solution.
- Represent the problem using written, oral, pictorial, or concrete representations.

Materials
- Discovering Math: Problem Solving video
- Computer with Internet access
- Print resources about a variety of weather phenomena (blizzard, hurricane, tornado, etc.)
- Pencil-and-Eraser Problem Activity Sheet (see below)
- Weather Phenomenon Activity Sheet (see below)
Procedures

1. Distribute copies of the Problem-Solving Activity Sheet. Review the problem-solving strategies students can use: divide a complex problem into smaller problems, consider a simpler problem, or work backwards.
   - Have students work with a partner to solve the problem. They should identify the problem-solving strategies and mathematical symbols used and explain their solution.

2. Tell students they will use the problem-solving strategies and critical-thinking skills presented in the video. Review the following concepts: approaches to solving a problem (make a table, create a model, and draw a diagram), framing problems and solutions (formulate a problem, collect necessary information, explain the solution), generalizations, conjectures, and inductive and deductive reasoning.

Students will choose a weather phenomenon (blizzard, hurricane, or tornado) to research. Based on their findings, they will make a generalization, a conjecture, and identify and solve a problem related to the chosen weather phenomenon. They will represent the solution in a table, diagram, or model and then present the solution orally, in writing, pictorially, or concretely.
   - Have students work in pairs. Each pair must choose a weather phenomenon to research. Using print or Internet resources they should identify a problem caused by the weather phenomenon. Then make a generalization based on their research (e.g., There was extreme flooding in every hurricane I researched so I can generalize that there is extreme flooding when a hurricane hits an area.).
   - Based on the generalization, have students make at least one conjecture. (e.g., If a mechanism is developed to stop the flow of water, the flood damage from a hurricane can be minimized.)
   - They should formulate (state and explain) one problem they will focus on (e.g., What type of mechanism can be developed to control or minimize flooding caused by a hurricane?).
   - Have students work with their partner to develop a solution to the problem. They may design a safety mechanism, form of communication, or other device to solve the problem.
   - They should represent their solution in a table, diagram, or model, as well as justify the strategy they chose (e.g., I used a diagram because it shows the way the mechanism was created. The diagram shows how all the steps are connected and related to each other.).
   - Have students identify limitations of their solutions.
   - Review the different ways to present a solution (written, oral, pictorial, or concrete). Students should decide how to present their solution to the class and justify their decision (e.g., I used a model so my classmates had a three-dimensional version of the mechanism that can be modified or manipulated.).
   - Have students present their solutions to the class.
Assessment

Use the following three-point rubric to evaluate students’ work during this lesson.

- **3 points:** Students correctly solved the pencil-and-eraser problem by identifying an economical solution and clearly explained the solution using mathematical language and symbols; clearly demonstrated the ability to research a weather phenomenon and identify and formulate a problem; clearly demonstrated the ability to make generalizations and conjectures; developed a reasonable and logical solution using effective problem-solving strategies; clearly explained and presented solution using effective presentation tools.

- **2 points:** Students satisfactorily solved the pencil-and-eraser problem by identifying a solution and explained the solution using some mathematical language and symbols; satisfactorily demonstrated the ability to research a weather phenomenon and identify and formulate a problem; satisfactorily demonstrated the ability to make generalizations and conjectures; developed a somewhat reasonable and logical solution using problem-solving strategies; satisfactorily explained and presented solution using presentation tools.

- **1 point:** Students incorrectly solved the pencil-and-eraser problem or did not explain the solution using mathematical language and symbols; did not demonstrate the ability to research a weather phenomenon and identify and formulate a problem; did not demonstrate the ability to make generalizations and conjectures; developed an unreasonable and illogical solution using ineffective problem-solving strategies; did not explain or present solution using effective presentation tools.

Vocabulary

**conjecture**

*Definition:* a statement, opinion, or conclusion based on guesswork
*Context:* The scientists made conjectures about climate and species.

**deductive reasoning**

*Definition:* reasoning that involves using known principles to draw conclusions about something specific
*Context:* Paula used deductive reasoning to conclude that most hurricanes cause extreme flooding.

**diagram**

*Definition:* a tool used to approach a problem that shows connections and relationships between separate pieces of information and makes hidden information more obvious
*Context:* The engineers used a diagram to plan the construction of a new bridge.

**formulate**

*Definition:* state definitely and clearly
*Context:* The teacher asked the students to formulate a problem.
**inductive reasoning**  
*Definition:* reasoning that involves observing, recognizing patterns and connections, and proposing possible explanations or generalizations  
*Context:* The student used inductive reasoning to conclude that the sum of the angles in all triangles equals 180 degrees.

**limitation**  
*Definition:* restraints that affect the solution to a problem  
*Context:* Since Oliver had only two weeks to complete the project, time was a limitation.

**model**  
*Definition:* a proportional, three-dimensional physical representation of an object  
*Context:* The students created a model of the car to show its features and proportions.

**table**  
*Definition:* a tool used to display and compare data  
*Context:* Jen created a table to display the data she collected on energy usage in California and Utah.

**Academic Standards**

Mid-continent Research for Education and Learning (McREL)  
McREL’s Content Knowledge: A Compendium of Standards and Benchmarks for K–12 Education addresses 14 content areas. To view the standards and benchmarks, visit [http://www.mcrel.org/compendium/browse.asp](http://www.mcrel.org/compendium/browse.asp).

This lesson plan addresses the following benchmarks:

- Understands how to break a complex problem into simpler parts or use a similar problem type to solve a problem.
- Uses a variety of strategies to understand problem-solving situations and processes.
- Formulates a problem, determines information required to solve the problem, chooses methods for obtaining this information, and sets limits for acceptable solutions.
- Represents problem situations in and translates among oral, written, concrete, pictorial, and graphical forms.
- Generalizes from a pattern of observations made in particular cases, makes conjectures, and provides supporting arguments for these conjectures.
- Constructs informal logical arguments to justify reasoning processes and methods of solutions to problems.
- Understands the role of written symbols in representing mathematical ideas and the use of precise language in conjunction with the special symbols of mathematics.
- Uses a variety of reasoning processes to model and to solve problems.
National Council of Teachers of Mathematics (NCTM)
The National Council of Teachers of Mathematics (NCTM) has developed national standards to provide guidelines for teaching mathematics. To view the standards online, go to http://standards.nctm.org.

This lesson plan addresses the following standards:

- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Monitor and reflect on the process of mathematical problem solving.
- Recognize reasoning and proof as fundamental aspects of mathematics.
- Make and investigate mathematical conjectures.
- Develop and evaluate mathematical arguments and proofs.
- Select and use various types of reasoning and methods of proof.

Support Materials

Develop custom worksheets, educational puzzles, online quizzes, and more with the free teaching tools offered on the Discoveryschool.com Web site. Create and print support materials, or save them to a Custom Classroom account for future use. To learn more, visit

- http://school.discovery.com/teachingtools/teachingtools.html

DVD Content

This program is available in an interactive DVD format. The following information and activities are specific to the DVD version.

How to Use the DVD

The DVD starting screen has the following options:

Play Video—This plays the video from start to finish. There are no programmed stops, except by using a remote control. With a computer, depending on the particular software player, a pause button is included with the other video controls.

Video Index—Here the video is divided into chapters indicated by title. Each chapter is then divided into four sections indicated by video thumbnail icons; brief descriptions are noted for each section. To play a particular segment, press Enter on the remote for TV playback; on a computer, click once to highlight a thumbnail and read the accompanying text description and click again to start the video.
Quiz — Each chapter has four interactive quiz questions correlated to each of the chapter’s four sections.

Standards Link — Selecting this option displays a single screen that lists the national academic standards the video addresses.

Teacher Resources — This screen gives the technical support number and Web site address.

Video Index

I. Problem Solving Using Simpler or Similar Problems (10 min.)

Using Simpler or Similar Problems: Introduction
People use problem-solving strategies to prepare for hurricanes. Solve a large problem by breaking it into smaller parts, researching historical data, or studying similar problems.

Example 1: Divide a Problem Into Simpler Sub-Problems
Mountain climbers divide a large problem into smaller, more manageable problems, such as preparation, supplies, and schedules.

Example 2: Divide a Complex Problem Into Simpler Sub-Problems
Air traffic controllers divide the complex problem of monitoring all air traffic into smaller, more manageable problems.

Example 3: Solve a Problem by Considering Similar Ones
Doctors and nurses solve problems by considering similar problems. They base diagnosis and treatment on previous cases.

II. Various Approaches to Problems (8 min.)

Various Approaches to Problems: Introduction
Creating a table, model, or diagram are effective strategies for solving a problem. Each can be used to organize information, see relationships, and find solutions.

Example 1: Make a Table
Tables effectively and efficiently display, analyze, and compare data. An example shows data comparing the energy sources used by China and the United States.

Example 2: Make A Model
Engineers and architects use models to help understand size, envision features, and identify advantages, disadvantages, and potential problems of future structures. Measurements, ratios, and proportions are used to create these models.

Example 3: Draw a Diagram
Diagrams show the connections between pieces of information, make hidden information more obvious, and solve problems. Architects use diagrams to see how phases of a project are related.
III. Framing Problems and Solutions (9 min.)

Framing Problems and Solutions: Introduction
There are three important parts in solving a problem: figuring out a way to formulate the problem; determining the information needed to solve the problem; and identifying the limitations of possible solutions.

Example 1: Formulate a Problem
Formulating a problem involves stating and explaining the problem and affects the way it is solved. Discover how NASA formulated a Mars landing.

Example 2: Determine and Collect Information
Identifying and collecting relevant and necessary information is a critical step in the problem-solving process. Information scientists gathered to determine if water existed on Mars models necessary data.

Example 3: Frame the Solution
Identifying solution limitations is a critical part of problem solving and affects the chosen solution. Explore the limitations of NASA’s Rover Project.

IV. Various Representations of Problem Situations (10 min.)

Various Representations of Problem Situations: Introduction
Problem situations represented orally, pictorially, or concretely are modeled and explained.

Example 1: Oral and Written Representations
Oral and written representations of problem situations occur when information is communicated verbally or in writing. Examples are news reports, newspaper, and magazine articles.

Example 2: Model and Concrete Representation
Scientists create models and concrete representations of objects to solve problems. The most important characteristic of such representations is that they can be modified or manipulated.

Example 3: Pictorial Representation Involving Graphs and Maps
Maps and graphs are types of pictorial representations used to solve problems. Weather forecasters use pictorial representations to communicate and study weather phenomena.

V. Generalization (9 min.)

Generalization: Introduction
Inductive reasoning involves observing, recognizing patterns and connections, and proposing possible explanations. Generalizations derive from inductive reasoning.

Example 1: Generalization From Observations
Developing general theories from specific observations is inductive reasoning. The theory of continental drift is an example.
Example 2: Conjecture From Generalization and Testing
Conjectures, hypotheses based on a generalization, are developed based on gathered information and can support a generalization. Conjectures about species and climate were made to support the theory of continental drift.

Example 3: Testing the Conjecture Further
Tectonic plates are identified as the mechanism that moved the continents. Scientists continue to make conjectures based on observations until a solution is found.

VI. Justification of Reasoning and Methods (8 min.)

Justification of Reasoning and Methods: Introduction
An inference is a generalization built from data. Generalizations develop from observations. Current generalizations and hypothesizes can change based on new observations and tests.

Example 1: Generalization From Observations
Observations are used to make generalizations. Aristotle used information from his observations to make the generalization that the Earth was spherical.

Example 2: Informal Deductions
Copernicus deduced from his observations that Earth and the other planets move around the sun. Galileo’s observations supported Copernicus’s theory.

Example 3: Universal Deductions
Hubble used his powerful telescope to observe the universe. He deduced that the universe is constantly expanding and that it is not uniform and infinite. Deductions are made from observations.

VII. Symbols (7 min.)

Symbols: Introduction
Symbols are objects or signs that represent an object or idea, such as traffic lights and signs and team uniforms and colors. Symbols in math represent quantities, operations and relationships, and variables.

Example 1: Symbols for Quantities
Quantities are represented by symbols and can also be symbolized by placement, such as in a clock or ruler.

Example 2: Symbols for Operations and Relationships
Symbols can represent operations or relationships between numbers. Common math symbols represent addition, subtraction, multiplication, division, and greater than, less than, or equal to.

Example 3: Symbols for Variables
Variables are symbols that represent specific quantities. The formula for finding area, using variables, is used as a model.
VIII. Various Methods of Solving Problems (8 min.)

**Various Methods of Solving Problems: Introduction**
Inductive reasoning is used when principles are developed based on patterns seen during observations. Deductive reasoning relies on known principles to draw conclusions about something specific.

**Example 1: Inductive Reasoning**
Inductive reasoning uses specific examples to develop a general rule, such as showing that the sum of the angles of any triangle equals 180 degrees.

**Example 2: Deductive Reasoning**
Deductive reasoning uses known principles to draw conclusions about something specific. It is used to deduce that the higher a diver jumps, the faster he will hit the water.

**Example 3: Combining Methods**
Working backward is a strategy that starts with the result and ends at the initial condition. Working backward, proportionality, and inductive and deductive reasoning help locate a lost ship.

**Quiz**

I. Problem Solving Using Simpler or Similar Problems

1. Which strategy would NOT be effective when solving a large problem?
   A. Study similar problems and solutions.
   B. Divide the problem into smaller parts.
   C. Research past solutions to similar problems.
   D. Study past solutions to problems that are not similar.

   **Answer: D**

2. John wants to ride his bike from Washington, D.C., to Chicago, Illinois. The total distance is 710 miles. He will begin on Monday, and ride 75 miles each weekday and 55 miles each weekend day. How many days will it take?
   A. 8
   B. 9
   C. 10
   D. 12

   **Answer: C**
3. A pilot is flying from San Francisco to San Diego, California. The total distance is 500 miles. The plane will be monitored by control towers during the flight, and there is a tower every 100 miles. If the flight begins at a control tower in San Francisco and ends at a control tower in San Diego, how many towers will monitor the plane during the flight?
   A. 4
   B. 5
   C. 6
   D. 7

   Answer: C

4. Doctors and nurses treat new cases by considering similar problems from past experiences. What would a doctor think if a patient complains of severe pain in an arm after falling off a chair?
   A. the patient has a broken leg
   B. the patient has a broken arm
   C. the patient has a severe cold
   D. the patient sprained their wrist

   Answer: B

II. Various Approaches to Problems

1. Lauren is researching the lengths of major U.S. rivers. What would be the most effective and efficient way for her to display the data so she could analyze and compare the lengths?
   A. create a model of one river
   B. create a diagram of one river
   C. create a diagram of each river
   D. create a table to display the length of each river

   Answer: D

2. Sara is an architect designing a new building. She needs to learn the size, envision the features, and identify potential advantages, disadvantages, and problems of the future building. What would be the most effective way for Sara to solve her problem?
   A. create a model of the building
   B. create a diagram of the building
   C. create a table to display the data about the building
   D. look at a diagram of a similar building that has already been built

   Answer: A
3. Walter is planning to build a new shed in his backyard. He wants to make a schedule to use during the construction process. What would be the most effective way for him to organize the information so he can see how each step connects to the next step?
   A. create a model of the shed
   B. look at a model of a similar shed
   C. create a table to display the information
   D. create a diagram to display the information

   Answer: D

III. Framing Problems and Solutions

1. The teacher asked the students to formulate the problems they are trying to solve. What does the teacher want the students to do?
   A. state and explain the problem
   B. gather information about the problem
   C. list the steps involved in solving the problem
   D. create a table to display the information from the problem

   Answer: A

2. Maggie is trying to design a sailboat that will travel faster than any other sailboat. What information is NOT relevant to her work and would NOT be necessary in solving the problem?
   A. the speed of the current fastest sailboat
   B. the speed of the wind when testing the sailboat
   C. the time she begins and ends working each day
   D. the weight of the materials she is planning to use

   Answer: C

3. Bruce must create a robot at least 24 inches tall that will water plants. Assuming the given information are the only parameters of the assignment, identify a limitation Bruce must consider when building the robot.
   A. budget
   B. time frame
   C. height of robot
   D. weight of robot

   Answer: C
IV. Various Representations of Problem Situations

1. The students in Ms. Kelly’s class were asked to report on the causes of tornadoes. Ms. Kelly would like an oral representation of the information. How should the students present their information?
   A. present a model of a tornado
   B. provide magazine and newspaper articles about tornadoes
   C. present pictures and drawings of tornadoes and their causes
   D. present the information by speaking

   **Answer: D**

2. Which type of problem-solving representation can be modified or manipulated to help identify a solution?
   A. oral representation
   B. written representation
   C. pictorial representation
   D. concrete representation

   **Answer: D**

3. Mark has a map that indicates the highest populated cities. He also has a graph comparing the populations of those cities. What type of representations does Mark have?
   A. oral representations
   B. written representations
   C. pictorial representations
   D. models

   **Answer: C**

V. Generalization

1. What is the name of the supercontinent suggested in the theory of continental drift?
   A. Pangaea
   B. Goepana
   C. Continent One
   D. Drifting Continent

   **Answer: A**
2. Identify a conjecture that supports the theory of continental drift.
   A. The continents once formed one supercontinent.
   B. Evidence of plants and animals should not be found in Antarctica.
   C. Evidence of changed climates should not be found on land that moved.
   D. There should be evidence of similar plants and animals at splitting points of continents.

   Answer: D

3. Identify the mechanism that moved the continents.
   A. tectonic plates
   B. volcanic plates
   C. mid-Atlantic ridge
   D. continental connectors

   Answer: A

VI. Justification of Reasoning and Methods

1. Jason observed a gas station for three weeks. He saw that people who own small cars use 10–12 gallons of gas to fill their tank, and people who own large cars use 18–20 gallons of gas. What generalization could Jason make based on his observations?
   A. People who own small cars drive faster.
   B. Large cars have smaller gas tanks than small cars.
   C. Small cars have smaller gas tanks than large cars.
   D. Gas tanks in small and large cars are the same size.

   Answer: C

2. What conjecture was made based on the observations of Copernicus and Galileo?
   A. The universe is finite.
   B. The sun revolves around the Earth.
   C. The universe is constantly expanding.
   D. The Earth and the other planets revolve around the sun.

   Answer: D

3. What conjecture was made based on Hubble’s observations?
   A. The universe is infinite.
   B. The universe is uniform.
   C. The universe is constantly expanding.
   D. The Earth is at the center of the universe.

   Answer: C
VII. Symbols

1. Identify the mathematical symbol that represents the quantity that is about 3.14
   A. \( \phi \)
   B. \( \pi \)
   C. \( e \)
   D. \( \rho \)
   
   Answer: B

2. Identify the mathematical symbol that correctly completes the number sentence.
   \( 24 \underline{\quad} 3 = 72 \)
   A. +
   B. -
   C. \( x \)
   D. >

   Answer: C

3. The formula for area is \( A = l \times w \)?
   The area of Yan’s living room is 192 square feet. If the length of the living room is 16 feet, what is the width?
   A. 12 feet
   B. 16 feet
   C. 20 feet
   D. 76 feet

   Answer: A

VIII. Methods of Solving Problems

1. In which example is Kim using inductive reasoning?
   A. Kim watches eight people dive. Each does a different dive. Kim concludes that the water is cold.
   B. Kim measures the length of 20 pinecones from the same tree. Each is about seven inches long. Kim concludes that the tree is 35 years old.
   C. Kim measures the temperature of 10 cups of water from the same cooler. Each is 60 degrees. Kim concludes that the water in the cooler is 60 degrees.
   D. Kim asks 40 people what they had for lunch: 30 hot dogs and 10 salads. Kim concludes that they were all on diets.

   Answer: C
2. In which example is Harry using deductive reasoning?
   A. Harry sees a sign that says haircuts are half priced on odd numbered dates. Harry deduces that there will be more people getting haircuts on the 17th than the 16th of the month.
   B. Harry measures the length of a hundred new pencils. Every pencil is 8 inches long. Harry deduces that new pencils vary in length.
   C. Harry asks 35 people how they get to work: 25 drive and 10 take the bus. Harry deduces that gas prices must be increasing.
   D. Harry measures the temperature of water from each faucet in the classroom. The temperature ranges 62-68 degrees. Harry deduces that the water from the faucets is cold.
   
   Answer: A

3. At the end of the day, Pat has 36 pencils on her desk. During the day, Kim gave her 12 pencils, John took 3 pencils, and Sue gave her 9 pencils. How many pencils did Pat have at the beginning of the day?
   A. 12
   B. 18
   C. 36
   D. 54
   
   Answer: B
Pencil-and-Eraser Problem Activity Sheet

The principal of Oak Town Middle School is buying pencils and erasers for all students. Each sixth grader will receive three pencils and four erasers, each seventh grader will receive two pencils and five erasers, and each eighth grader will receive two pencils and six erasers. There are 254 sixth graders, 289 seventh graders, and 312 eighth graders in the school. The principal needs to know how many boxes, crates, or pallets of pencils and erasers to buy so that each student receives the correct number. She wants to spend the least amount of money and have the smallest number of leftover supplies.

Use the information below to determine the most economical way to purchase the supplies.

<table>
<thead>
<tr>
<th><strong>Pencils</strong></th>
<th><strong>Erasers</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>There are 75 pencils in one box.</td>
<td>There are 115 erasures in one box.</td>
</tr>
<tr>
<td>There are three boxes in one crate.</td>
<td>There are four boxes in one crate.</td>
</tr>
<tr>
<td>There are five crates on one pallet.</td>
<td>There are four crates on one pallet.</td>
</tr>
<tr>
<td>Box of pencils = $3.65</td>
<td>Box of erasers = $2.95</td>
</tr>
<tr>
<td>Crate of pencils = $9.25</td>
<td>Crate of erasers = $10.50</td>
</tr>
<tr>
<td>Pallet of pencils = $40.75</td>
<td>Pallet of pencils = $38.75</td>
</tr>
</tbody>
</table>

Describe your solution to the problem. Use mathematical language and symbols to explain how you solved the problem. Be sure to explain how you determined the number of boxes, crates, or pallets of pencils and erasers to order, the total cost, and the number of leftover pencils and erasers.
Weather Phenomenon Activity Sheet

1. Choose a weather phenomenon to research.

2. Identify important information about the weather phenomenon.

3. Make a generalization and a conjecture about the weather based on your research.

4. Formulate the problem you will solve.

5. Work with your partner to develop a solution to the problem.

6. Identify and justify the problem-solving approaches you used.

7. Identify limitations of your solution.

8. Identify and justify a way to present your solution to the class.

9. Present your solution to the class.