Discovering Math
Concepts in Algebra: Ratios and Proportions
Teacher’s Guide

**Grade Level:** 10–12  **Curriculum Focus:** Mathematics  **Lesson Duration:** Three class periods

**Program Description**
How are ratios and proportions used to solve real world problems? This video features them in Renaissance design, calculating the orbit of the moon, estimating animal populations, understanding currency exchange rates, making distance-speed conversions, and creating special effects in films.

**Onscreen Questions**
- What are some ways to measure a large length or distance indirectly?
- How are similar figures used in blueprints and other plans?

Part II, “Math and Design in the Renaissance”
- How did Renaissance artists use math in their work?
- What is the golden ratio, and how was it used in art and architecture during the Renaissance?

**Lesson Plan**

**Student Objectives**
- Write a one-page paper describing how a Renaissance artist used mathematics to depict perspective.
- Apply principles of linear perspective to works of art.
- Learn how to construct the golden ratio.
- Learn how the golden ratio is used in art and architecture and how it relates to the Fibonacci sequence.
**Materials**

- *Discovering Math: Concepts in Algebra: Ratios and Proportions* video
- Computer with Internet access
- Calculators

**Procedures**

1. Have students watch the segment “Math and Design in the Renaissance” in *Discovering Math: Concepts in Algebra: Ratios and Proportions*. Tell them they will each choose a painter featured in the video (Filippo Brunelleschi, Leon Battista Alberti, or Piero della Francesca) to research and report how he used perspective in his art.

2. When students have completed their research, ask them to write a one-page paper.

3. Have each student choose a partner with whom they will share their papers and answer any questions. Then have students summarize their partners’ papers for the class, including at least three interesting facts.

4. Have students locate examples of Renaissance paintings that demonstrate linear perspective. From the Web sites below and other references, students may print out examples and identify the horizon line, the vanishing point, and the orthogonal lines.
   - Exploring Linear Perspective
     [www.renaissanceconnection.org/perspective.html](http://www.renaissanceconnection.org/perspective.html)
   - From Human Architecture to Architectural Structure
     [http://www.mcm.edu/academic/galileo/ars/arshtml/arch2.html](http://www.mcm.edu/academic/galileo/ars/arshtml/arch2.html)
   - Leonardo da Vinci: Mathematical Order in Art
     [http://www.facstaff.bucknell.edu/udaep/090/w2/Magee.htm](http://www.facstaff.bucknell.edu/udaep/090/w2/Magee.htm)

5. Have students recreate the construction of Alberti’s tiled floor.

6. Review the process of determining the golden ratio by dividing a segment of length 1 into two pieces of lengths $x$ and $1-x$, where $\frac{1}{x} = \frac{x}{1-x}$. Have students use a $72^\circ$, $72^\circ$, $36^\circ$ isosceles triangle to construct the golden ratio.

7. Review Fibonacci’s rabbit problem and draw the first three levels of the family tree. Allow students time to extend the tree.

8. Show students a recursive formula for the Fibonacci sequence: $f_1=1$, $f_2=1$, $f_n=f_{n-2} + f_{n-1}$.
9. Point out that the first pattern shown (i.e., \( f_1 + f_3 = f_4, f_1 + f_3 + f_5 = f_6, f_1 + f_3 + f_5 + f_7 = f_8 \)) works only starting with \( f_1 \). Challenge students to find a pattern for the addition of alternate elements starting with \( f_2 \) (i.e., \( f_2 + f_4, f_2 + f_4 + f_6, f_2 + f_4 + f_6 + f_8, \) etc.) They should find that \( f_2 + f_4 = f_3 - 1, f_2 + f_4 + f_6 = f_7 - 1, \) etc.

10. Have students research patterns occurring in the Fibonacci sequence. The following Web sites are a good starting point. Have students write a one-page paper showing at least two different patterns and share their patterns with classmates.
   - Fibonacci numbers
   - The Fibonacci numbers
     [http://math.holycross.edu/~davids/fibonacci/fibonacci.html](http://math.holycross.edu/~davids/fibonacci/fibonacci.html)
   - Fibonacci Numbers and the Golden Section
     [http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fib.html](http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fib.html)

11. Have students research the construction of a golden rectangle to construct a golden rectangle and the associated spiral. The following Web sites provide helpful information.
   - Golden Ratio, Fibonacci Sequence
   - Fibonacci in Nature
     [http://www.geom.uiuc.edu/~demo5337/s97b/spiral.html](http://www.geom.uiuc.edu/~demo5337/s97b/spiral.html)
   - Activities
     [http://cuip.uchicago.edu/~dlnarain/golden/activities.htm](http://cuip.uchicago.edu/~dlnarain/golden/activities.htm)

12. Review the derivation of the golden ratio from the Fibonacci sequence algebraically.

**Assessment**

Use the following three-point rubric to evaluate students’ work during this lesson.

- **3 points:** Students wrote complete papers with all the requested information; clearly demonstrated the ability to apply principles of linear perspective to art works and to relate the Fibonacci sequence to the golden ratio; and properly constructed a golden ratio.
• **2 points:** Students wrote papers with most the requested information; demonstrated some ability to apply principles of linear perspective to art works and to relate the Fibonacci sequence to the golden ratio; and constructed a golden ratio.

• **1 point:** Students wrote incomplete papers; could not apply principles of linear perspective to art works or relate the Fibonacci sequence to the golden ratio; and could not construct a golden ratio.

**Vocabulary**

**Fibonacci sequence**  
*Definition:* A sequence in which each term is the sum of the two terms immediately preceding it  
*Context:* The Fibonacci sequence begins 1, 1, 2, 3, 5…

**golden ratio**  
*Definition:* A number arising from dividing a line segment in such a way that the ratio of the whole segment to the larger piece is equal to the ratio of the larger piece to the smaller piece  
*Context:* Rectangles with sides constructed to satisfy the golden ratio are considered the most esthetically pleasing.

**orthogonal lines**  
*Definition:* Lines that result when perspective is used to represent parallel lines on a flat surface  
*Context:* Sets of orthogonal lines meet at the same point on the horizon line.

**perspective**  
*Definition:* The process of drawing objects on a flat surface to accurately represent their depth and size  
*Context:* The development of perspective transformed art during the Italian Renaissance.

**similar figures**  
*Definition:* Figures with the same shape but not necessarily the same size  
*Context:* Scientists use similar figures to analyze objects that are too large or too small to measure directly.

**Academic Standards**

**National Council of Teachers of Mathematics (NCTM)**  
The National Council of Teachers of Mathematics provides guidelines for teaching mathematics in grades K–12 to promote mathematical literacy. To view the standards, visit this Web site:  
This lesson plan addresses the following thematic standards:

- Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture.
- Generalize patterns using explicitly defined and recursively defined functions.

**Mid-continent Research for Education and Learning (McREL)**

McREL’s Content Knowledge: A Compendium of Standards and Benchmarks for K-12 Education addresses 14 content areas. To view the standards and benchmarks, visit [http://www.mcrel.org/compendium/browse.asp](http://www.mcrel.org/compendium/browse.asp).

This lesson plan addresses the following national standards:

- Mathematics: Uses proportionality to model problems; understands the mathematical concepts of similarity and congruence; uses discrete structures (e.g., finite graphs, matrices, sequences) to represent and solve problems.
- History: Understands how European society experienced cultural transformations in an age of global intercommunication between 1450 and 1750.
- Art: Understands and applies media, techniques, and processes related to the visual arts.

**Support Materials**

Develop custom worksheets, educational puzzles, online quizzes, and more with the free teaching tools offered on the Discoveryschool.com Web site. Create and print support materials, or save them to a Custom Classroom account for future use. To learn more, visit [http://school.discovery.com/teachingtools/teachingtools.html](http://school.discovery.com/teachingtools/teachingtools.html)

**DVD Content**

This program is available in an interactive DVD format. The following information and activities are specific to the DVD version.

**How To Use the DVD**

The DVD starting screen has the following options:

*Play Video*—This plays the video from start to finish. There are no programmed stops, except by using a remote control. With a computer, depending on the particular software player, a pause button is included with the other video controls.
Video Index—Here the video is divided into sections indicated by video thumbnail icons; brief descriptions are noted for each one. Watching all parts in sequence is similar to watching the video from start to finish. To play a particular segment, press Enter on the remote for TV playback; on a computer, click once to highlight a thumbnail and read the accompanying text description and click again to start the video.

Curriculum Units—These are specially edited video segments pulled from different sections of the video (see below). These nonlinear segments align with key ideas in the unit of instruction. They include onscreen pre- and post-viewing questions, reproduced below in this Teacher’s Guide. Total running times for these segments are noted. To play a particular segment, press Enter on the TV remote or click once on the Curriculum Unit title on a computer.

Standards Link—Selecting this option displays a single screen that lists the national academic standards the video addresses.

Teacher Resources—This screen gives the technical support number and Web site address.

Video Index

I. Conversions and Ratios: Dollars to Drams (5 min.)
Learn how ratios and rates are used and apply them to find conversions among global currencies.

II. Estimating and Proportions: Counting Sheep (4 min.)
Find out the purpose of the capture-recapture method and learn how proportions are used in it.

III. Similar Figures: Scaled Down (5 min.)
Define and analyze similar figures. Learn figures on a blueprint relate to their real-life counterparts, and use ratios to find lengths on blueprints and in reality.

IV. Indirect Measurement: High in the Sky (4 min.)
Learn methods of indirect calculation and see how scientists use triangulation to calculate large distances. Apply trigonometry and indirect measurement techniques to find the distance to stars.

V. Math and Design in the Renaissance (28 min.)
Learn how Renaissance painters relied on mathematics. Discover the relationship of the golden to art and architecture and to the Fibonacci sequence.

Curriculum Units

I. Exchange Rates
Pre-viewing question
Q. Why do countries have different currencies?
A. Answers will vary.

Post-viewing question
Q. If one U.S. dollar is worth 6.24 Danish kroner, how many dollars is one krone worth?
A. \( \frac{1}{6.24} \approx 0.16 \). A Danish krone is worth approximately 0.16 dollars.

II. The Capture-Recapture Method
Pre-viewing question
Q. Name an endangered or extinct species.
A. Answers will vary.

Post-viewing question
Q. A wildlife researcher tags 60 giraffes in an isolated population. Later she examines 100 giraffes and finds that 15 of them have tags. Estimate the giraffe population.
A. \( \frac{M}{N} = \frac{m}{n} \). \( \frac{60}{15} = \frac{60(100)}{15} = 400 \)

III. Blueprints and Similar Figures
Pre-viewing question
Q. Name two objects that have the same shape but different sizes.
A. Answers will vary.

Post-viewing question
Q. A kitchen is 12 feet long and 18 feet wide. If it is drawn 3 inches long on a blueprint, how wide is the kitchen on the blueprint?
A. \( \frac{12 \text{ feet}}{3 \text{ inches}} = \frac{18 \text{ feet}}{x \text{ inches}} \). \( x = \frac{(18 \text{ feet})(3 \text{ inches})}{(12 \text{ feet})} = 4.5 \text{ inches} \)

IV. Indirect Measurement and Trigonometry
Pre-viewing question
Q. Name something too large or too small to measure directly.
A. Answers will vary.

Post-viewing question
Q. Jim is standing next to a flagpole. Jim is 5 feet tall and casts a shadow 8 feet long. The flagpole casts a shadow 40 feet long; how tall is it?
A. \( \frac{x \text{ feet}}{40 \text{ feet}} = \frac{5 \text{ feet}}{8 \text{ feet}} \). \( x = 25 \). The flagpole is 25 feet tall.

V. Measuring Stars
Pre-viewing question
Q. Why is it useful to know the distance of a star?
A. Answers will vary.

Post-viewing question
Q. What does the term “parallax angle” mean?
A. It is the angle of the apparent shift in a star’s position as viewed with more distant stars.

VI. The Renaissance
Pre-viewing question
Q. Give the name of a Renaissance artist.
A. Answers will vary.

Post-viewing question
Q. What invention caused the increased production and distribution of books beginning in the 15th century?
A. The printing press

VII. Linear Perspective
Pre-viewing question
Q. How do artists use geometry?
A. Answers will vary.

Post-viewing question
Q. Explain how the vanishing point and the horizon line are used in paintings.
A. Answers will vary.

VIII. Brunelleschi and Alberti
Pre-viewing question
Q. What are parallel lines?
A. Answers will vary.

Post-viewing question
Q. In Alberti’s construction, what is the significance of the distance from Z, the vanishing point, to X, the point chosen in line with Z but outside the square?
A. It is the viewing distance, or the distance from the painter to the picture.

IX. Piero’s Theorem
Pre-viewing question
Q. How can you prove that two triangles are similar?
A. Answers will vary.

Post-viewing question
Q. What is Piero’s Theorem?
A. If lines are drawn from a point through two parallel lines, they will be divided in the same proportion.
X. The Golden Ratio
Pre-viewing question
Q. What is the quadratic formula?
A. \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \), where \( x \) is given by the quadratic equation \( ax^2 + bx + c = 0 \).

Post-viewing question
Q. What is the golden ratio \( \varphi \) to three decimal places?
A. 1.618.

XI. The Fibonacci Sequence
Pre-viewing question
Q. What is a sequence?
A. Answers will vary.

Post-viewing question
Q. What is the Fibonacci sequence?
A. In this sequence the first two terms are 1 and 1, and every subsequent term is the sum of the two terms preceding it.

XII. The Golden Ratio in the Fibonacci Sequence
Pre-viewing question
Q. What value does the sequence \( 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots \) approach after the sequence has been continued for many terms?
A. 0

Post-viewing question
Q. Why do \( x_n \) and \( x_{n-1} \) approach the same value as \( n \) becomes very large?
A. Answers will vary.

XIII. The Golden Ratio in Nature
Pre-viewing question
Q. Where is math found in nature?
A. Answers will vary.

Post-viewing question
Q. Give two natural settings in which the golden ratio is found.
A. Answers will vary.